Within Subjects Designs

Psy 420

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Topics in WS designs

- Types of Repeated Measures Designs
- Issues and Assumptions
- Analysis
  - Traditional One-way
  - Regression One-way

Within Subjects?

- Each participant is measured more than once
- Subjects cross the levels of the IV
- Levels can be ordered like time or distance
- Or levels can be un-ordered (e.g. cases take three different types of depression inventories)

Within Subjects?

- WS designs are often called repeated measures
- Like any other analysis of variance, a WS design can have a single IV or multiple factorial IVs
  - E.g. Three different depression inventories at three different collection times

Within Subjects?

- Repeated measures designs require less subjects (are more efficient) than BG designs
  - A 1-way BG design with 3 levels that requires 30 subjects
    - The same design as a WS design would require 10 subjects
  - Subjects often require considerable time and money, it's more efficient to use them more than once

Within Subjects?

- WS designs are often more powerful
  - Since subjects are measured more than once we can better pin-point the individual differences and remove them from the analysis
  - In ANOVA anything measured more than once can be analyzed, with WS subjects are measured more than once
  - Individual differences are subtracted from the error term, therefore WS designs often have substantially smaller error terms

Types of WS designs

- Time as a variable
  - Often time or trials is used as a variable
  - The same group of subjects are measured on the same variable repeatedly as a way of measuring change
  - Time has inherent order and lends itself to trend analysis
  - By the nature of the design, independence of errors (BG) is guaranteed to be violated

Types of WS designs

- Matched Randomized Blocks
    1. Measure all subjects on a variable or variables
    2. Create “blocks” of subjects so that there is one subject in each level of the IV and they are all equivalent based on step 1
    3. Randomly assign each subject in each block to one level of the IV

Issues and Assumptions

- Big issue in WS designs
  - Carryover effects
    - Are subjects changed simple by being measured?
    - Does one level of the IV cause people to change on the next level without manipulation?
    - Safeguards need to be implemented in order to protect against this (e.g. counterbalancing, etc.)

Issues and Assumptions

- Normality of Sampling Distribution
  - In factorial WS designs we will be creating a number of different error terms, may not meet +20 DF
  - Than you need to address the distribution of the sample itself and make any transformations, etc.
  - You need to keep track of where the test for normality should be conducted (often on combinations of levels)
  - Example

Issues and Assumptions

- Independence of Errors
  - This assumption is automatically violated in a WS design
  - A subject’s score in one level of the IV is automatically correlated with other levels, the close the levels are (e.g. in time) the more correlated the scores will be.
  - Any consistency in individual differences is removed from what would normally be the error term in a BG design

Issues and Assumptions

- Sphericity
  - The assumption of Independence of errors is replaced by the assumption of Sphericity when there are more than two levels
  - Sphericity is similar to an assumption of homogeneity of covariance (but a little different)
  - The variances of difference scores between levels should be equal for all pairs of levels

Issues and Assumptions

- Sphericity
  - The assumption is most likely to be violated when the IV is time
    - As time increases levels closer in time will have higher correlations than levels farther apart
    - The variance of difference scores between levels increase as the levels get farther apart

Issues and Assumptions

- Additivity
  - This assumption basically states that subjects and levels don't interact with one another
  - We are going to be using the A x S variance as error so we are assuming it is just random
  - If A and S really interact than the error term is distorted because it also includes systematic variance in addition to the random variance

Issues and Assumptions

- Additivity
  - The assumption is literally that difference scores are equal for all cases
  - This assumes that the variance of the difference scores between pairs of levels is zero
  - So, if additivity is met than sphericity is met as well
  - Additivity is the most restrictive assumption but not likely met

Issues and Assumptions

- Compound Symmetry
  - This includes Homogeneity of Variance and Homogeneity of Covariance
  - Homogeneity of Variance is the same as before (but you need to search for it a little differently)
  - Homogeneity of Covariance is simple the covariances (correlations) are equal for all pairs of levels.

Issues and Assumptions

- If you have additivity or compound symmetry than sphericity is met as well
  - In additivity the variance are 0, therefore equal
  - In compound symmetry, variances are equal and covariances are equal
- But you can have sphericity even when additivity or compound symmetry is violated (don’t worry about the details)
- The main assumption to be concerned with is sphericity

Issues and Assumptions

- Sphericity is usually tested by a combination of testing homogeneity of variance and Mauchly's test of sphericity (SPSS)
  - If violated (Mauchly's), first check distribution of scores and transform if non-normal; then recheck.
  - If still violated…

Issues and Assumptions

- If sphericity is violated:
  1. Use specific comparisons instead of the omnibus ANOVA
  2. Use an adjusted F-test (SPSS)
     - Calculate degree of violation (ε)
     - Then adjust the DFs downward (multiplies the DFs by a number smaller than one) so that the F-test is more conservative
     - Greenhouse-Geisser, Huynh-Feldt are two approaches to adjusted F (H-F preferred, more conservative)

Issues and Assumptions

- If sphericity is violated:
  3. Use a multivariate approach to repeated measures (take Psy 524 with me next semester)
  4. Use a maximum likelihood method that allows you to specify that the variance-covariance matrix is other than compound symmetric (don't worry if this makes no sense)

Analysis – 1-way WS

- Example

	A: Month			Case average
	a_1	a_2	a_3	
	$Month_1$	$Month_2$	$Month_3$	
s_1	1	3	6	**3.333**
s_2	1	4	8	**4.333**
s_3	3	3	6	**4.000**
s_4	5	5	7	**5.667**
s_5	2	4	5	**3.667**
Mean	**2.4**	**3.8**	**6.4**	**GM = 4.2**
SD	**1.673**	**0.837**	**1.140**	

Analysis – 1-way WS

- The one main difference in WS designs is that subjects are repeatedly measured.
- Anything that is measured more than once can be analyzed as a source of variability
- So in a 1-way WS design we are actually going to calculate variability due to subjects
- So, $SS_T = SS_A + SS_S + SS_{A \times S}$
- We don't really care about analyzing the SS_S but it is calculated and removed from the error term

Sums of Squares

- The total variability can be partitioned into Between Groups (e.g. measures), Subjects and Error Variability

$$\sum\left(Y_{ij}-\bar{Y}_{..}\right)^2=\sum n_j\left(\bar{Y}_{.j}-\bar{Y}_{..}\right)^2+g\sum\left(\bar{Y}_{i.}-\bar{Y}_{..}\right)^2+$$

$$+\left[\sum\left(Y_{ij}-\bar{Y}_{.j}\right)^2-g\sum\left(\bar{Y}_{i.}-\bar{Y}_{..}\right)^2\right]$$

$$SS_{Total}=SS_{BetweenGroups}+SS_{Subjects}+SS_{Error}$$

$$SS_T=SS_{BG}+SS_S+SS_{Error}$$

Analysis – 1-way WS

- Traditional Analysis
  - In WS designs we will use s instead of n
  - $DF_T = N - 1$ or $as - 1$
  - $DF_A = a - 1$
  - $DF_S = s - 1$
  - $DF_{A \times S} = (a - 1)(s - 1) = as - a - s + 1$

  - Same drill as before, each component goes on top of the fraction divided by what's left
  - 1s get T^2/as and "as" gets ΣY^2

Analysis – 1-way WS

- Computational Analysis - Example

	A: Month			Case Totals
	a_1	a_2	a_3	
	$Month_1$	$Month_2$	$Month_3$	
s_1	1	3	6	**10**
s_2	1	4	8	**13**
s_3	3	3	6	**12**
s_4	5	5	7	**17**
s_5	2	4	5	**11**
Treatment Totals	**12**	**19**	**32**	**T = 63**

Analysis – 1-way WS

- Traditional Analysis

$$SS_A = \frac{\sum\left(\sum A_j\right)^2}{s} - \frac{T^2}{as}$$

$$SS_S = \frac{\sum\left(\sum S_i\right)^2}{a} - \frac{T^2}{as}$$

$$SS_{AxS} = \sum Y^2 - \frac{\sum\left(\sum A_j\right)^2}{s} - \frac{\sum\left(\sum S_i\right)^2}{a} + \frac{T^2}{as}$$

$$SS_T = \sum Y^2 - \frac{T^2}{as}$$

Analysis – 1-way WS

- Traditional Analysis - Example

$$SS_A = \frac{12^2 + 19^2 + 32^2}{5} - \frac{63^2}{3(5)} = \frac{1,529}{5} - \frac{3,969}{15} = 305.8 - 264.6 = 41.2$$

$$SS_S = \frac{10^2 + 13^2 + 12^2 + 17^2 + 11^2}{3} - 264.6 = \frac{823}{3} - 264.6 = 274.3 - 264.6 = 9.7$$

$$SS_{AxS} = 325 - 305.8 - 274.3 + 264.6 = 9.5$$

$$SS_T = 325 - 264.6 = 60.4$$

Analysis – 1-way WS

- Traditional Analysis - example

Source	SS	df	MS	F
A	41.20	2	20.60	17.35
S	9.70	4	2.43	2.04
AxS	9.50	8	1.19	
Total	60.40	14		

Analysis – 1-way WS

- Regression Analysis
  - With a 1-way WS design the coding through regression doesn't change at all concerning the IV (A)
  - You need a – 1 predictors to code for A
  - The only addition is a column of sums for each subject repeated at each level of A to code for the subject variability

			A		S			
		Y	X_1	X_2	X_3	YX_1	YX_2	YX_3
	S_1	1						
	S_2	1						
a_1	S_3	3						
	S_4	5						
	S_5	2						
	S_1	3						
	S_2	4						
a_2	S_3	3						
	S_4	5						
	S_5	4						
	S_1	6						
	S_2	8						
a_3	S_3	6						
	S_4	7						
	S_5	5						
		63						
		325						

			A		S			
		Y	X_1	X_2	X_3	YX_1	YX_2	YX_3
	S_1	1	-1					
	S_2	1	-1					
a_1	S_3	3	-1					
	S_4	5	-1					
	S_5	2	-1					
	S_1	3	0					
	S_2	4	0					
a_2	S_3	3	0					
	S_4	5	0					
	S_5	4	0					
	S_1	6	1					
	S_2	8	1					
a_3	S_3	6	1					
	S_4	7	1					
	S_5	5	1					
		63	0					
		325	10					

			A		S			
		Y	X_1	X_2	X_3	YX_1	YX_2	YX_3
	S_1	1	-1	1				
	S_2	1	-1	1				
a_1	S_3	3	-1	1				
	S_4	5	-1	1				
	S_5	2	-1	1				
	S_1	3	0	-2				
	S_2	4	0	-2				
a_2	S_3	3	0	-2				
	S_4	5	0	-2				
	S_5	4	0	-2				
	S_1	6	1	1				
	S_2	8	1	1				
a_3	S_3	6	1	1				
	S_4	7	1	1				
	S_5	5	1	1				
		63	0	0				
		325	10	30				

			A		S			
		Y	X_1	X_2	X_3	YX_1	YX_2	YX_3
	S_1	1	-1	1	10			
	S_2	1	-1	1	13			
a_1	S_3	3	-1	1	12			
	S_4	5	-1	1	17			
	S_5	2	-1	1	11			
	S_1	3	0	-2	10			
	S_2	4	0	-2	13			
a_2	S_3	3	0	-2	12			
	S_4	5	0	-2	17			
	S_5	4	0	-2	11			
	S_1	6	1	1	10			
	S_2	8	1	1	13			
a_3	S_3	6	1	1	12			
	S_4	7	1	1	17			
	S_5	5	1	1	11			
		63	0	0	189			
		325	10	30	2469			

			A		S	SP		
		Y	X_1	X_2	X_3	YX_1	YX_2	YX_3
	S_1	1	-1	1	10	-1	1	10
	S_2	1	-1	1	13	-1	1	13
a_1	S_3	3	-1	1	12	-3	3	36
	S_4	5	-1	1	17	-5	5	85
	S_5	2	-1	1	11	-2	2	22
	S_1	3	0	-2	10	0	-6	30
	S_2	4	0	-2	13	0	-8	52
a_2	S_3	3	0	-2	12	0	-6	36
	S_4	5	0	-2	17	0	-10	85
	S_5	4	0	-2	11	0	-8	44
	S_1	6	1	1	10	6	6	60
	S_2	8	1	1	13	8	8	104
a_3	S_3	6	1	1	12	6	6	72
	S_4	7	1	1	17	7	7	119
	S_5	5	1	1	11	5	5	55
		63	0	0	189	20	6	823
		325	10	30	2469			

Analysis – 1-way WS

- Regression Analysis

$$SS_Y = \sum Y^2 - \frac{(\sum Y)^2}{N} = 325 - \frac{63^2}{15} = 60.4$$

$$SS(X_1) = \sum X_1^2 - \frac{(\sum X_1)^2}{N} = 10 - \frac{0^2}{15} = 10$$

$$SS(X_2) = \sum X_2^2 - \frac{(\sum X_2)^2}{N} = 30 - \frac{0^2}{15} = 30$$

$$SS(X_3) = \sum X_3^2 - \frac{(\sum X_3)^2}{N} = 2469 - \frac{189^2}{15} = 87.6$$

Analysis – 1-way WS

- Regression Analysis

$$SP(YX_1) = \sum YX_1 - \frac{(\sum Y)(\sum X_1)}{N} = 20 - \frac{(63)(0)}{15} = 20$$

$$SP(YX_2) = \sum YX_2 - \frac{(\sum Y)(\sum X_2)}{N} = 6 - \frac{(63)(0)}{15} = 6$$

$$SP(YX_3) = \sum YX_3 - \frac{(\sum Y)(\sum X_3)}{N} = 823 - \frac{(63)(189)}{15} = 29.2$$

Analysis – 1-way WS

- Regression Analysis

$$SS_A = SS(\text{reg. } X_1) + SS(\text{reg. } X_2) = \frac{[SP(YX_1)]^2}{SS(X_1)} + \frac{[SP(YX_2)]^2}{SS(X_2)} = \frac{(20)^2}{10} + \frac{(6)^2}{30} = 41.2$$

$$SS_S = SS(\text{reg. } X_3) = \frac{[SP(YX_3)]^2}{SS(X_3)} = \frac{(29.2)^2}{87.6} = 9.7$$

$$SS_T = SS_Y = 60.4$$

$$SS(\text{resid.}) = SS_T - SS_A - SS_S = 60.4 - 41.2 - 9.7 = 9.5$$

Analysis – 1-way WS

- Regression Analysis
  - Degrees of Freedom
    - df_A = # of predictors = a – 1
    - df_S = # of subjects – 1 = s – 1
    - df_T = # of scores – 1 = as – 1
    - $df_{AS} = df_T - df_A - df_S$