

Psy 420 Andrew Ainsworth

Topics in WS designs

- Types of Repeated Measures Designs
- Issues and Assumptions
- Analysis
 - Traditional One-way
 - Regression One-way

- Each participant is measured more than once
- Subjects cross the levels of the IV
- Levels can be ordered like time or distance
- Or levels can be un-ordered (e.g. cases take three different types of depression inventories)

- WS designs are often called repeated measures
- Like any other analysis of variance, a WS design can have a single IV or multiple factorial IVs
 - E.g. Three different depression inventories at three different collection times

- Repeated measures designs require less subjects (are more efficient) than BG designs
 - A 1-way BG design with 3 levels that requires 30 subjects
 - The same design as a WS design would require 10 subjects
 - Subjects often require considerable time and money, it's more efficient to use them more than once

• WS designs are often more powerful

- Since subjects are measured more than once we can better pin-point the individual differences and remove them from the analysis
- In ANOVA anything measured more than once can be analyzed, with WS subjects are measured more than once
- Individual differences are subtracted from the error term, therefore WS designs often have substantially smaller error terms

Types of WS designs

- Time as a variable
 - Often time or trials is used as a variable
 - The same group of subjects are measured on the same variable repeatedly as a way of measuring change
 - Time has inherent order and lends itself to trend analysis
 - By the nature of the design, independence of errors (BG) is guaranteed to be violated

Types of WS designs

- Matched Randomized Blocks
 - 1. Measure all subjects on a variable or variables
 - Create "blocks" of subjects so that there is one subject in each level of the IV and they are all equivalent based on step 1
 - 3. Randomly assign each subject in each block to one level of the IV

- Big issue in WS designs
 - Carryover effects
 - Are subjects changed simple by being measured?
 - Does one level of the IV cause people to change on the next level without manipulation?
 - Safeguards need to be implemented in order to protect against this (e.g. counterbalancing, etc.)

- Normality of Sampling Distribution
 - In factorial WS designs we will be creating a number of different error terms, may not meet +20 DF
 - Than you need to address the distribution of the sample itself and make any transformations, etc.
 - You need to keep track of where the test for normality should be conducted (often on combinations of levels)
 - Example

- Independence of Errors
 - This assumption is automatically violated in a WS design
 - A subject's score in one level of the IV is automatically correlated with other levels, the close the levels are (e.g. in time) the more correlated the scores will be.
 - Any consistency in individual differences is removed from what would normally be the error term in a BG design

• Sphericity

- The assumption of Independence of errors is replaced by the assumption of Sphericity when there are more than two levels
- Sphericity is similar to an assumption of homogeneity of covariance (but a little different)
- The variances of difference scores between levels should be equal for all pairs of levels

• Sphericity

- The assumption is most likely to be violated when the IV is time
 - As time increases levels closer in time will have higher correlations than levels farther apart
 - The variance of difference scores between levels increase as the levels get farther apart

• Additivity

- This assumption basically states that subjects and levels don't interact with one another
- We are going to be using the A x S variance as error so we are assuming it is just random
- If A and S really interact than the error term is distorted because it also includes systematic variance in addition to the random variance

- Additivity
 - The assumption is literally that difference scores are equal for all cases
 - This assumes that the variance of the difference scores between pairs of levels is zero
 - So, if additivity is met than sphericity is met as well
 - Additivity is the most restrictive assumption but not likely met

Compound Symmetry

- This includes Homogeneity of Variance and Homogeneity of Covariance
- Homogeneity of Variance is the same as before (but you need to search for it a little differently)
- Homogeneity of Covariance is simple the covariances (correlations) are equal for all pairs of levels.

- If you have additivity or compound symmetry than sphericity is met as well
 - In additivity the variance are 0, therefore equal
 - In compound symmetry, variances are equal and covariances are equal
- But you can have sphericity even when additivity or compound symmetry is violated (don't worry about the details)
- The main assumption to be concerned with is sphericity

- Sphericity is usually tested by a combination of testing homogeneity of variance and Mauchly's test of sphericity (SPSS)
 - If violated (Mauchly's), first check distribution of scores and transform if non-normal; then recheck.
 - If still violated...

- If sphericity is violated:
 - 1. Use specific comparisons instead of the omnibus ANOVA
 - 2. Use an adjusted F-test (SPSS)
 - Calculate degree of violation (ε)
 - Then adjust the DFs downward (multiplies the DFs by a number smaller than one) so that the Ftest is more conservative
 - Greenhouse-Geisser, Huynh-Feldt are two approaches to adjusted F (H-F preferred, more conservative)

- If sphericity is violated:
 - 3. Use a multivariate approach to repeated measures (take Psy 524 with me next semester)
 - Use a maximum likelihood method that allows you to specify that the variancecovariance matrix is other than compound symmetric (don't worry if this makes no sense)

o Example

		A: Month)	
	a ₁	a ₂	a ₃	Case
	$Month_1$	Month ₂	Month ₃	average
S ₁	1	3	6	3.333
S ₂	1	4	8	4.333
S ₃	3	3	6	4.000
S ₄	5	5	7	5.667
S 5	2	4	5	3.667
Mean	2.4	3.8	6.4	GM = 4.2
SD	1.673	0.837	1.140	GIVI – 4.Z

• • Analysis – 1-way WS

- The one main difference in WS designs is that subjects are repeatedly measured.
- Anything that is measured more than once can be analyzed as a source of variability
- So in a 1-way WS design we are actually going to calculate variability due to subjects

• So, $SS_T = SS_A + SS_S + SS_{A \times S}$

 We don't really care about analyzing the SS_S but it is calculated and removed from the error term

• • Sums of Squares

 The^Itotal variability can be partitioned into Between Groups (e.g. measures), Subjects and Error Variability

$$\sum \left(Y_{ij} - \overline{Y}_{..}\right)^{2} = \sum n_{j} \left(\overline{Y}_{.j} - \overline{Y}_{..}\right)^{2} + g \sum \left(\overline{Y}_{i.} - \overline{Y}_{..}\right)^{2} + \left[\sum \left(Y_{ij} - \overline{Y}_{.j}\right)^{2} - g \sum \left(\overline{Y}_{i.} - \overline{Y}_{..}\right)^{2}\right]$$
$$SS_{Total} = SS_{BetweenGroups} + SS_{Subjects} + SS_{Error}$$
$$SS_{T} = SS_{BG} + SS_{S} + SS_{Error}$$

• Traditional Analysis

In WS designs we will use s instead of n

•
$$DF_T = N - 1$$
 or as -1

•
$$DF_{A \times S} = (a - 1)(s - 1) = as - a - s + 1$$

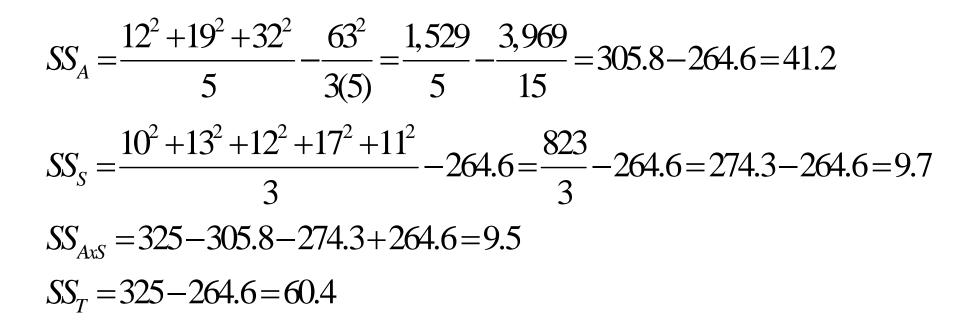
- Same drill as before, each component goes on top of the fraction divided by what's left
- 1s get T²/as and "as" gets ΣY^2

• Computational Analysis - Example

		A: Month			
	a ₁	a ₂	a ₃	Case Totals	
	$Month_1$	$Month_2$	$Month_3$	TOLAIS	
s ₁	1	3	6	10	
S ₂	1	4	8	13	
S ₃	3	3	6	12	
S ₄	5	5	7	17	
S ₅	2	4	5	11	
Treatment Totals	12	19	32	T = 63	

 Traditional Analysis $SS_A = \frac{\sum \left(\sum A_j\right)^2}{s} - \frac{T^2}{as}$ $SS_{S} = \frac{\sum \left(\sum S_{i}\right)^{2}}{T^{2}}$ as Ω $SS_{AxS} = \sum Y^2 - \frac{\sum \left(\sum A_j\right)^2}{\sum \left(\sum S_i\right)^2} - \frac{\sum \left(\sum S_i\right)^2}{\sum \left(\sum S_i\right)^2} + \frac{T^2}{\sum \left(\sum$ as $SS_T = \sum Y^2 - \frac{T^2}{T}$

• Traditional Analysis - Example



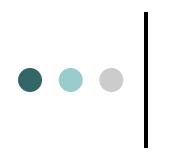
• Traditional Analysis - example

Source	SS	df	MS	F
Α	41.20	2	20.60	17.35
S	9.70	4	2.43	2.04
AxS	9.50	8	1.19	
Total	60.40	14		

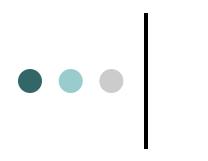
Analysis – 1-way WS

Regression Analysis

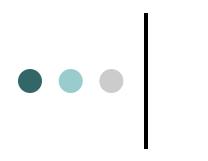
- With a 1-way WS design the coding through regression doesn't change at all concerning the IV (A)
- You need a 1 predictors to code for A
- The only addition is a column of sums for each subject repeated at each level of A to code for the subject variability



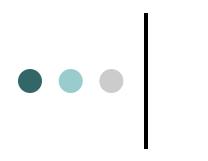
			А	S	
		Y	$X_1 X_2$	X ₃	$YX_1 YX_2 YX_3$
	S_1	1			
	S_2	1			
a_1	S_3	3			
	S_4	5			
	S_5	2			
	S_1	3			
	S_2	4			
a_2	S_3	3			
	S_4	5			
	S_5	4			
	S_1	6			
	S_2	8			
a_3	S_3	6			
	S_4	7			
	S_5	5			
		63			
		325	5		



			Α	S	
		Y	$X_1 X_2$	X_3	$YX_1 YX_2 YX_3$
	S_1	1	-1		
	S_2	1	-1		
a_1	S_3	3	-1		
	S_4	5	-1		
	S_5	2	-1		
	S_1	3	0		
	S_2	4	0		
a_2	S_3	3	0		
	S_4	5	0		
	S_5	4	0		
	S_1	6	1		
	S_2	8	1		
a_3	S_3	6	1		
	S_4	7	1		
	S_5	5	1		
		63	0		
		325	10		



				A	S	
		Y	X_1	X_2	X_3	$YX_1 YX_2 YX_3$
	S_1	1	-1	1		
	S_2	1	-1	1		
a_1	S_3	3	-1	1		
	S_4	5	-1	1		
	S_5	2	-1	1		
	S_1	3	0	-2		
	S_2	4	0	-2		
a_2	S_3	3	0	-2		
	S_4	5	0	-2		
	S_5	4	0	-2		
	S_1	6	1	1		
	S_2	8	1	1		
a_3	S_3	6	1	1		
	S_4	7	1	1		
	S_5	5	1	1		
		63	0	0		
		325	10	30		



				4	S	
		Y	X_1	X_2	X_3	$YX_1 YX_2 YX_3$
	S_1	1	-1	1	10	
	S_2	1	-1	1	13	
a_1	S_3	3	-1	1	12	
	S_4	5	-1	1	17	
	S_5	2	-1	1	11	
	S_1	3	0	-2	10	
	S_2	4	0	-2	13	
a_2	S_3	3	0	-2	12	
	S_4	5	0	-2	17	
	S_5	4	0	-2	11	
	S_1	6	1	1	10	
	S_2	8	1	1	13	
a_3	S_3	6	1	1	12	
	S_4	7	1	1	17	
	S_5	5	1	1	11	
		63	0	0	189	
		325	10	30	2469	



						Ą	S		SP	
				Y	X ₁	X_2	X ₃	$\mathbf{Y}\mathbf{X}_1$	YX_2	YX_3
			S_1	1	-1	1	10	-1	1	10
			S_2	1	-1	1	13	-1	1	13
		a_1	S_3	3	-1	1	12	-3	3	36
			S_4	5	-1	1	17	-5	5	85
			S_5	2	-1	1	11	-2	2	22
			S_1	3	0	-2	10	0	-6	30
			S_2	4	0	-2	13	0	-8	52
		a_2	S_3	3	0	-2	12	0	-6	36
			S_4	5	0	-2	17	0	-10	85
			S_5	4	0	-2	11	0	-8	44
			S_1	6	1	1	10	6	6	60
			S_2	8	1	1	13	8	8	104
		a_3	S_3	6	1	1	12	6	6	72
			S_4	7	1	1	17	7	7	119
			S_5	5	1	1	11	5	5	55
				63	0	0	189	20	6	823
				325	10	30	2469			

Regression Analysis

$$SS_{Y} = \sum Y^{2} - \frac{(\sum Y)^{2}}{N} = 325 - \frac{63^{2}}{15} = 60.4$$

$$SS(X_{1}) = \sum X_{1}^{2} - \frac{(\sum X_{1})^{2}}{N} = 10 - \frac{0^{2}}{15} = 10$$

$$SS(X_{2}) = \sum X_{2}^{2} - \frac{(\sum X_{2})^{2}}{N} = 30 - \frac{0^{2}}{15} = 30$$

$$SS(X_{3}) = \sum X_{3}^{2} - \frac{(\sum X_{3})^{2}}{N} = 2469 - \frac{189^{2}}{15} = 87.6$$

$$SP(YX_1) = \sum YX_1 - \frac{(\sum Y)(\sum X_1)}{N} = 20 - \frac{(63)(0)}{15} = 20$$
$$SP(YX_2) = \sum YX_2 - \frac{(\sum Y)(\sum X_2)}{N} = 6 - \frac{(63)(0)}{15} = 6$$
$$SP(YX_3) = \sum YX_3 - \frac{(\sum Y)(\sum X_3)}{N} = 823 - \frac{(63)(189)}{15} = 29.2$$

$$SS_{A} = SS(reg. X_{1}) + SS(reg. X_{2}) = \frac{[SP(YX_{1})]^{2}}{SS(X_{1})} + \frac{[SP(YX_{2})]^{2}}{SS(X_{2})} = \frac{(20)^{2}}{10} + \frac{(6)^{2}}{30} = 41.2$$

$$SS_{S} = SS(reg. X_{3}) = \frac{[SP(YX_{3})]^{2}}{SS(X_{3})} = \frac{(29.2)^{2}}{87.6} = 9.7$$

$$SS_{T} = SS_{Y} = 60.4$$

$$SS(resid.) = SS_{T} - SS_{A} - SS_{S} = 60.4 - 41.2 - 9.7 = 9.5$$

Analysis – 1-way WS

Regression Analysis

Degrees of Freedom

•
$$df_A = #$$
 of predictors = $a - 1$

•
$$df_s = #$$
 of subjects $-1 = s - 1$

•
$$df_{AS} = df_T - df_A - df_S$$